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the means of converting any distance, say an inch, on the photograph very exactly into are; so that if we measure the distance between the two points and convert it in to are we have the second polar coordinate. This conversion can be made if we know the distance from the surface of the lens to the surface of the sensitive plate, just as we find the angular value of a micrometer screw by measuring the focal distance of the objective and counting the number of threads of the screw to an inch. The distance between the surfaces has been measured with an accuracy that seems much within the limits required. We have therefore the second polar coordinate and the solution is theoretically complete.

But in order to establish the usefulness of the photographic method it remains to be shown that the photographs can be made in distant places transported thousands of miles through great change of temperature, that the collodion film is subject to no contraction or change except such as can be fully accounted for in the measurement and calculation, and finally that their linear measurement can be made with the required accuracy. We may easily get some idea of the accuracy required in the measurements of the photographs. The sun's diameter is about 32 minutes of arc, and the photographs being 4 inches in diameter we shall have $\frac{1}{1000}$ of an inch equal to $0''.48$, and $\frac{1}{10000}$ of an inch equal to $0''.05$. The photographs to be measured are the negatives on the glass plates, and on these Venus appears as a round vacant spot $\frac{1}{8}$ of an inch in diameter. As this spot is a symmetrical one, and generally well defined it is probable that the pointings on this spot can be made with sufficient accuracy. The difficulty will be in fixing the position of the sun's center. This can be done only by proceeding from the edge of the photograph, and the edge, or limb, of the sun is a very difficult thing to deal with, on account of its inequalities. Here perhaps some assistance may be derived from the systems of ruled lines, which are designed to control any contraction of the collodion film, and which may be used as a system of rectangular coordinates to which the centers of the sun and Venus may be referred.

We have now at the Naval Observatory a number of photographs from different stations, and shall probably soon know with what accuracy they can be measured.

SOLUTIONS OF PROBLEMS.

Solutions of problems in No. 2 have been received as follows:

From J. M. Arnold, 63; A. L. Baker, 61 & 65; W. W. Beman, 59, 60,

61, 63, 65 & 66; Marcus Baker, 59, 60, 61, 65, 66 & 67; Prof. P. E. Chase, 59, 60, 61 & 65; Geo. M. Day, 59, 60, 61, 63, 64, & 65; Dr. H. Eggers, 62 & 63; E. S. Farrow, 59, 60, 61, 63, 64, 65, 66 & 67; Henry Gunder, 59, 60, 61, 63, 64, 65 & 66; Wm. Hoover, 59, 60, 61, 63, 65 & 66; G. W. Hill, 67; Artemas Martin, 59, 60, 61, 63, 64, 65 & 66; J. B. Mott, 61; F. P. Matz, 59 & 60; E. P. Norton, 63; O. D. Oathout, 59, 60 & 65; L. Regan, 59, 60, 61, 62 & 63; E. B. Seitz, 59, 60, 61, 63, 64 & 65; Walter Siverly, 59, 60, 61, 62 & 65; E. T. T., 59 & 60; J. S. Mays, 59 to 66.

59.—“Find a number, consisting of two places of figures, whose half squared will equal the number inverted.”

SOLUTION BY PRES. E. T. TAPPAN, KENYON COLLEGE, GAMBIER, OHIO.

Problem 59 gives this equation $[\frac{1}{2}(10x + y)]^2 = 10y + x$, $\therefore y = 20 - 10x \pm \sqrt{(400 - 396)}$. Evidently the only integral value of x is 1. Then $y = 8$.

SOLUTION BY PROF. PLINY EARLE CHASE, PHILADELPHIA, PA.

Let $2x$ be the number sought. Then, in accordance with the Arabic system of numeration, $x^2 - 2x = x(x - 2) = 9y$. Since x , the larger factor, can contain only one figure, if $2x$ is an integer y must be the smaller factor. $\therefore x = 9$; $y = 7$; $2x = 18$. If $2x$ is decimal, its right-hand digit must be 0, and the square of half the tenths digit must be one tenth of the digit. $\therefore x^2 = .2x$; $x = .20$; $y = -1.8$; $2x = .40$.

[Because the same digits that represent $2x$ also represent x^2 ; therefore the excess of 9's contained in the sum of the digits is the same in both cases; consequently the excess of 9's in $x^2 - 2x$ is zero. $\therefore x^2 - 2x$ is a multiple of 9, equal $9y$ say.—Ed.]

60.—“Find a value for x that will make $\frac{62x + 1984}{x^2 - 1024}$ a whole number.”

SOLUTION BY PRES. E. T. TAPPAN.

The given expression reduces to $\frac{62}{x - 32}$, which is a whole number when $x - 32$ is a measure of 62. Hence x may be 94, 63, $52\frac{2}{3}$, $47\frac{1}{2}$, &c.; 35, 33 &c.

61.—“What will be the value of each letter of the alphabet if the product of all but a is 1, all but b is 2, all but c is 3, and so on to all but z is 26.”

SOLUTION BY PROF. P. E. CHASE.

Let p = the product. Then

$$\frac{p}{a} = \frac{p}{2b} = \frac{p}{3c} = \dots \frac{p}{26z} \quad p = a = 2b = 3c = \dots 26z = \frac{a^{26}}{2 \cdot 3 \cdot 4 \dots 26}.$$

$$a = \sqrt[26]{(2 \cdot 3 \cdot 4 \dots 26)} = 11.59375; \quad b = 5.79688; \quad c = 3.86459; \quad \&c.$$

62.—“Given four lines in a plane: To inscribe a parallelogram within them with given direction of sides.”

SOLUTION BY DR. H. EGGERS, MILWAUKEE, WISCONSIN.

The four given right lines may be named a, b, c, d , and the two given directions p and q .

Take on the line a an arbitrary point P , draw from P a line parallel to p , till it intersects line b in Q ; from Q draw a line parallel to q , intersecting line c in R ; from R draw a line parallel to p , intersecting line d in S ; from S draw a line parallel to q intersecting line a in a point M . Now if the points P and M should coincide, the problem would be solved. But as this circumstance in general will not occur, take on the line a another arbitrary point P' and proceed exactly as above, till a second point M' results on line a , analogous to point M : Now take a third arbitrary point P'' on line a and proceeding in the same manner, construct a third point M'' on line a . The two systems of points on line a , viz; P, P', P'' and M, M', M'' will form two homographic systems of points on line a . Find now the two coincident points of these two systems of points, which can be done by one fixed arbitrary circle and the ruler alone. These two points may be X and Y ; then every one will furnish one solution of the problem. For we have only to repeat the above trial construction, beginning with point X or Y , instead of the arbitrary point P ; then the fourth line connecting d with a will pass through point X or Y respectively.

If The succession of the four lines a, b, c, d , is fixed, the number of solutions will be in general two. But if all possible combinations of the four given lines are admissible, we find 12 different combinations, consequently $12 \times 2 = 24$ solutions.

[Mr Siverly's solution of this question is by Analytical Geometry, and is brief and elegant. Mr. J. S. Hays, of Hodgenville, Ky., sent a very ingenious and elegant geometrical construction and demonstration; his diagram is larger however than we can, consistently, admit to our pages; and we exact of correspondents, when diagrams are necessary to illustrate their subjects, that they be correctly drawn, suitable for the engraver to copy, and not to exceed three inches in their largest dimension; and always less when practicable.]

63.—Given the sides AB , AC and BC , and the angles α and β , (see diagram below), to find the sides AD , BD and CD .

SOLUTION BY W. W. BEMAN, UNIVERSITY OF MICH., ANN ARBOR, MICH.

Construction. Construct the triangle ABC . Upon AB construct a segment containing an angle of 24° , and upon BC a segment containing an angle of 16° . The intersection of these arcs will determine the point D , the distance of which from A , B and C can now be measured.

It is evident that there will be two solutions, one when B and D are on the same side of AC , and another when they are on opposite sides.

Trigonometrical Computation. Designate the angles and sides as in the figure.

Then $\alpha + \beta + x + y + B = 360^\circ$, or
 $x + y = 360^\circ - (\alpha + \beta + B)$. From

ABD , $BD = \frac{c \sin x}{\sin \alpha}$; from BDC , BL

$$= \frac{\alpha \sin y}{\sin \beta} \quad \therefore \frac{c \sin x}{\sin \alpha} = \frac{\alpha \sin y}{\sin \beta}, \text{ or}$$

$$\frac{\sin x}{\sin y} = \frac{\alpha \sin \alpha}{c \sin \beta} = \tan \phi \text{ (say).}$$

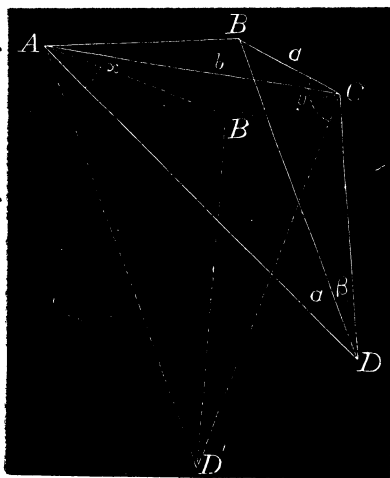
$$\begin{aligned} \text{Then } \frac{\sin x + \sin y}{\sin x - \sin y} &= \frac{\tan \frac{1}{2}(x + y)}{\tan \frac{1}{2}(x - y)} \\ &= \frac{\tan \phi + 1}{\tan \phi - 1} = \tan \left(\phi + \frac{1}{4}\pi \right). \end{aligned}$$

Hence we can find x and y .

$$BD = \frac{c \sin x}{\sin \alpha} = 18.88, \quad AD = \frac{c \sin ABD}{\sin \alpha} = 23.65, \quad CD = \frac{a \sin DBC}{\sin \beta} = 15.17 \text{ ms.}$$

When B and D are on the same side of AC , $x + y = B - (\alpha + \beta)$, but the remainder of the work is the same.

This method of computation is known as Delambre's.



64.—“Find the maximum value of $\left(\frac{a}{x}\right)^x$ without the aid of the Calculus”.

SOLUTION BY ARTEMAS MARTIN, ERIE, PA.

The given function will be a maximum when its Napierian logarithm is a maximum. $\therefore u = x \log a - x \log x$,

$$\text{and} \quad u' = (x + h) \log a - (x + h) \log (x + h)$$

$$= (x + h) \log a - (x + h) \log x - \left(x + h \right) \left(\frac{h}{x} \right) + \frac{1}{2} \left(x + h \right) \left(\frac{h}{x} \right)^2 - \&c.$$

The coefficient of h is $\log a - \log x - 1$, which put $= 0$ and we have $x = \frac{a}{e}$; $\therefore$ the maximum value of the given function is $e^{\frac{a}{e}}$.

[This question may be solved by a simpler process as follows:

The given expression in logarithms is $x \log a - x \log x = a \max., = x \log m$. Dividing by x , $\log a - \log x = \log m$; whence $x = a \div m$, and the original expression becomes $m^{\frac{a}{m}} = a \max.$, or $a \cdot \frac{\log m}{m} = a \max.$ But $\frac{\log m}{m}$ is a maximum when m is the base of the system. Therefore $m = e$, and the required maximum is $a \cdot \frac{\log e}{e} = \frac{a}{e^e}$. —Ed.]

65.—“The corner of a page is turned down, and in every position the area of the triangle is two square inches; find the locus of the angular point.”

SOLUTION BY WALTER SIVERLY, OIL CITY, PA.

Let r and θ be the polar coordinates of the locus referred to the corner before it is turned down and one edge of the leaf. The double area of the triangle $= \frac{1}{2}r(\tan \theta + \cot \theta) = 4$, or $r^2 = 16 \sin \theta \cos \theta = 8 \sin 2\theta$ is the polar equation of the curve, which is therefore the lemniscate.

66.—“ A speaks the truth b times out of a ; B , d times out of c ; and C , n times out of m .

C says that B told him that A said a certain event transpired. Required the probability that the event occurred.”

[Artemas Martin gets for answer to this question,

$$\frac{bdn}{acm} + \frac{b}{a}\left(1 - \frac{d}{c}\right)\left(1 - \frac{n}{m}\right) + \frac{d}{c}\left(1 - \frac{b}{a}\right)\left(1 - \frac{n}{m}\right) + \frac{n}{m}\left(1 - \frac{b}{a}\right)\left(1 - \frac{d}{c}\right);$$

and for authority he refers to Todhunter's History of the Theory of Probability, p. 462; and Todhunter's Algebra, 4th edition, p. 464. E. S. Farrow gets the same result as Mr. Martin. All the other correspondents give for the required chance $bdn \div acm$.

If we assume that all the witnesses did actually testify *in relation to the event*, then we think there is no doubt that Mr Martin's result is correct:

For, let it be assumed that the event did in fact occur; and suppose, in a given statement, that C tells the truth, and that B , in informing C , told a lie. Then B must have told C that A said the event occurred, therefore, because B lied, A must have told B that the event did not occur; but by hypothesis the event *did* occur, therefore A lied. Hence if C asserts that an event has occurred, and if his statement is based on the fact that B has testified to him in relation to the event, whose testimony in turn is based on A 's statement in relation to the event; then it follows that, if any two in making their statements tell a lie and the other one tells the truth, the event must have occurred.

We contend, however, that the question, as announced, does not warrant the inference that all three necessarily testified in relation to the event; and that when *C* says that *B* said that *A* said a certain thing, if *C* lies, he may either lie in relation to the *character* of the statement which he reports, or he may lie in the affirmation that *B* made a statement; and there is no reason assigned from which we can infer that he would be more likely to lie in respect to the character of the statement than in relation to the fact of the statement. But if we admit that *C* lied in attributing a statement to *B*, when no statement of any kind in relation to the event was made by *B*; and that *B* in like manner lied in attributing a statement to *A*, when no statement was made by *A*; then it is clear that the chance in favor of the event resulting from the concurrence of *C* and *B* in a lie with *A* in the truth, is only one fourth what is assigned to that contingency above. And, for the same reason, the chance resulting from the concurrence of *C* and *A*, and *B* and *A* respectively in telling a lie, with the other one in telling the truth, is one half what is assigned to that contingency above. To warrant Mr. Martin's result we claim therefore that the statement of the question must be modified.

Mr. Martin has appended to his solution of 66, a note correcting a misprint in Todhunter's *History of the Theory of Probability*, which we give below.—Ed.]

NOTE, BY ARTEMAS MARTIN. — The denominator of the formula for *traditional* testimony given in *Todhunter's History of the Theory of Probability*, p. 462, third paragraph, is wrong—a misprint probably. I will quote a portion of the second and third paragraphs:

"Suppose a witness to speak truth m times and falsehood n times out of $m + n$ times; let m' and n' have similar meanings for a second witness. Then if they agree in an assertion the probability of its truth is

$$\frac{mm'}{mm' + nn'} \quad * * * *$$

Using the same notation as before if one witness reports a statement from the report of another the probability of its truth is

$$\frac{mm' + nn'}{(m + m')(n + n')};$$

for the statement is true if they both tell the truth or if they both tell a falsehood."—The latter formula should be

$$\frac{mm' + nn'}{(m + n)(m' + n')}.$$

[We are compelled, for want of room, to defer publishing the solution of problem 67 until the issue of the July No.]